Research Methodology
Part 8

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Statistical Power

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Statistical Power

- Statistical power
  - A statistical inference can go wrong, if there is insufficient statistical power
  - Statistical power is the measure of probability that a statistical test will reject a false null hypothesis
  - Higher the statistical power, more likelihood is there to find statistical significance if the null hypothesis is false
  - Example:
    - The null hypothesis is that MD and DNB students study equal number of hours
    - Alternative hypothesis is that DNB students study more
    - A cohort study is set up with random samples from both groups to test the hypothesis

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- ***Contd. (Power)***
  - Mean study hours for MDs = 10
  - Mean study hours for DNBs = 10.45
  - Now mean scores are to be compared using t-test or ANOVA
  - The study result says that there is no difference
  - In reality there is a difference in study hours in both
  - In this case a type II error (β) has been made
  - Probability of doing this is defined as power ($1 - \beta$)
  - The reason could be that the study lacked sufficient statistical power to detect actual difference
  - It is accepted that studies should have 80 percent statistical power to avoid Type II errors

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- ***Contd. (Power)***
- What determines statistical power?
  - Mainly the statistical power is determined by:
    - The levels of significance or α (alpha)
      - Usual significance levels adopted are 0.1, 0.05, 0.01
    - Type I error or alpha (α)
      - Same symbol is used for both level of significance as well as Type I error
    - The magnitude of hypothesized effect size
      - i.e., the magnitude of the differences between group means or other test statistic
    - The standard deviation of hypothesised effect size
    - The size of the sample

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- More specifically power is a function of alpha and beta
  - As alpha increases, beta decreases correspondingly
    - This increases the power
- Also Power is a function of effect size
  - That is true alternative hypothesis such as an expected difference between groups
- Statistical power should be determined before the study begins
- This will determine the sample size necessary to obtain at least 80 percent (≥ 0.80) power
  - This size is based on other two factors, i.e., significance and effect size

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- ***Contd. (Power)***
- Power analysis is an essential component of research design
  - A study should be undertaken when a probability of finding an existing effect exists
    - This can be done when the sample size is large enough
  - If a sample is too small the study may not be able to detect the important differences
  - If the sample size is too large, the demand on respondents' time and privacy will be unreasonable

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- It will also waste scarce resources
- Power analysis has two important benefits:
  - Sample size can be determined rationally
  - A substantially interesting effect size can be specified
- Power analysis can be done by software programme such as SAS, Stata, S-Plus
- SPSS has a separate programme called SamplePower for calculating a sample size for a given power

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Estimation and Hypothesis Testing

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Estimation

- We really are interested in population and not in sample
- But population is usually inaccessible, or
  - it would be too costly and time consuming to reach every element of the population
- Therefore by studying various statistic of samples, we make an inference about the population
- There are two methods for making inference about the population:
  - Estimation
  - Hypothesis testing

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Estimation

- Concept of Estimation
  - Objective of estimation is:
    - To determine the approximate value of a population parameter
    - This is done on the basis of sample statistic
    - Example:
      - Sample mean is used to determine population mean
      - The sample value is called estimate
  - There are two types of estimators
    - Point Estimator
    - Interval estimator

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- Point estimator
  - It is the process by which a single value of a parameter of a population is estimated
  - This is done by using a sample statistic
  - Disadvantages of point estimator:
    - It is almost certain that the estimate will be wrong
    - The probability that $\bar{X}$ will exactly equal μ is almost 0
    - It is expected that the estimator should be close to the parameter
    - It may be assumed that an estimator derived from a large sample will produce result that is more accurate
    - But an estimator cannot reflect the effect of sample size

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- Interval estimator
  - Therefore, the interval estimator should be used for estimating population parameter
  - Interval estimator states that the parameter lies between two numbers
  - Interval estimator is affected by sample size
  - There are a large number of applications where estimation occurs
  - Example:
    - A TV network executive wants to know the proportion of television viewer who are tuned in to their programme
    - This can be done and drawing a sample and calculating mean
    - This sample mean can then be extrapolated to the population to draw a conclusion

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Hypothesis

- What is Hypothesis?
  - It is a conjectural statement of the relationship between two or more variables (Kerlinger)
  - It can also be defined as:
    - A proposition that is stated in a testable form and that predicts a particular relationship between two or more variables (Bailey)
  - Functions:
    - It draws attention to the specific aspects that is to be investigated
    - Delineates data collection
    - Enhances objectivity

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Constructing Hypothesis

- Characteristics of hypothesis
  - Should be clear and precise
  - Should be capable of being tested
  - Should state relationship between variables
  - Should be limited in scope but specific
  - Should be stated in simple terms and understandable
  - Should be consistent with most known facts
  - Should be amenable to testing within reasonable time
  - Must explain the facts that give rise to the need for explanation

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Constructing Hypothesis

- Types
  - Null hypothesis
    - It is a hypothesis of no difference
    - This is the starting point
    - It should be tested whether null hypothesis is likely to be true
  - Alternate hypothesis / Research hypothesis
    - It directly contradicts null hypothesis
    - It states that the actual value of a population parameter is less than, greater than, or not equal to the value stated in the null hypothesis
    - It is opposite of research hypothesis

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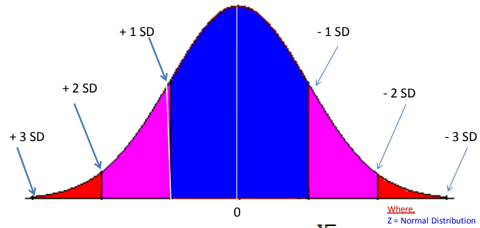
Normal Distribution



$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

Where,
Z = Normal Distribution Statistic
X″ = Sample distribution
μ = Population mean
n = Sample size
σ = (sigma) standard deviation

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Contd. (Normal curve …)

- Some Important Properties of Normal curve:
  1. Area under the curve = 1
  2. Finding the area under the curve
     a. To the left of any z value
        - Look up the z value in the table and use the area given
     b. To the right of any z value
        - Look up the z value and subtract from 1
     c. Between any two z values
        - Look up both z values and subtract the corresponding areas

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Hypothesis Testing

Concepts

- Concepts of hypothesis testing
  - The term Hypothesis Testing is new to most researchers
  - But the concepts underlying it are quite familiar
  - There are many non-statistical applications of hypothesis testing
  - One familiar example is trial of an accused in a court of law
  - The justice system considers the accused as innocent till proved otherwise

- The prosecution tries to prove that the accused is guilty
- The judge makes a decision on the basis of evidence presented
- In fact, the judge conducts a test of hypothesis
- There are actually two hypotheses that are tested
  - The first is called **Null Hypothesis**
    - This is represented by H_0

- ***Contd. (hypothesis …)***
  - H_0: The defendant is innocent
- The second is called the **alternative hypothesis** (or **research hypothesis**)
  - It is denoted as H_1 or H_a
  - H_1: The defendant is guilty
- To begin with the judge does not know which hypothesis is correct
  - The judge must make decision on the basis of evidence presented by both prosecution and defense

- ***Contd. (Hypothesis ..)***
- There are only two possible decisions:
  - Convict or acquit the dependant
- Analogy can be drawn from this to hypothesis testing
  - Convicting the defendant is equivalent to *rejecting the null hypothesis in favor of the alternative hypothesis*
  - Acquitting a defendant is equivalent to *not rejecting the null hypothesis*
- In these expressions, it is never said that null hypothesis is accepted ➔ Innocence need not be proved

- ***Contd. (Errors …)***
- Errors
  - There are two possible errors
  - A **Type I error** occurs when we reject a true null hypothesis
  - A **Type II error** is defined as not rejecting a false null hypothesis
  - In the criminal trial, a **Type I** error is made when an innocent person is wrongly convicted
  - A **Type II** error occurs when a guilty dependant is acquitted

Hypothesis Testing

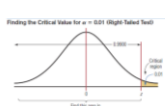
- Two methods
  1. Rejection Region Method
  2. P-value method
- Rejection Region Method
  - Steps:
    1. State the hypothesis and identify the claim
    2. Find the critical value(s) from the appropriate table
    3. Compute the test value
    4. Make decision to reject or not reject the null hypothesis

- *Contd. (Hypothesis …)*
- Step 1: Stating the hypothesis
  - Both null and alternative hypothesis to be stated
  - Example 1:
    - The VP Hospital operations (Ops) urges Medical Admin to ensure all discharges are declared before 1 PM
    - Medical Admin says that at least 98% discharges are declared by or before 1 PM
    - Ops claims it is not so, less than 98% are declared before 1 PM
    - Write the null and alternative hypothesis
      - H_0: $\mu \geq 98$ Or H_0: $\mu = 98$
      - H_1: $\mu < 98$
- The null hypothesis is always written as $H_0 = x$

- *Contd. (Hypothesis …)*
- Example 2:
  - As per NABH requirement, all case sheets to be countersigned by consultants within 24 hours
  - Quality Team (QT) says that it is done so
  - Medical admin claims it takes more than that time
  - Write the hypotheses
    - H_0: $\mu = 24$
    - H_1: $\mu > 24$
- Example 3:
  - In the above example if the medical admin says it is not equal to 24 hours, then the hypotheses are:
    - H_0: $\mu = 24$
    - H_1: $\mu \neq 24$

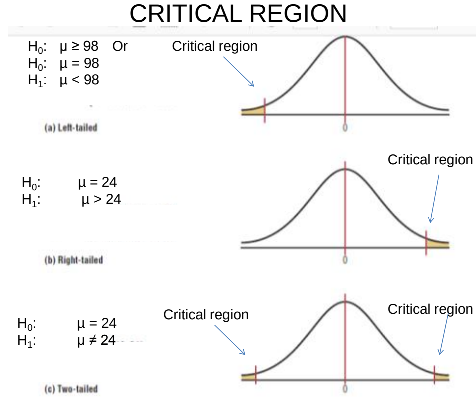
- *Contd. (Hypothesis …)*
- Step 2: Find the critical value
  - Critical value separates the critical region from the non-critical region (The symbol is C.V) in the bell curve
  - The critical or rejection region:
    - It is the range of values of the test value that indicates that there is a significant difference, and
      - that the null hypothesis should be rejected
  - The critical value can be on the right side of the mean or on the left side of the mean for a one-tailed test

- *Contd. (critical …)*
- The location of critical value depends on the inequality sign of the alternative hypothesis



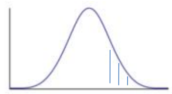
  - These signs are less than <, greater than >, or not equal to ≠
  - In example 1, H_1: $\mu < 98$, it will be a left tailed test
  - In example 2, H_1: $\mu > 24$, it will be a right tailed test
  - In example 3, H_1: $\mu \neq 24$, it will be a two tailed test
- For critical value, level of significance to be decided arbitrarily by the researcher
  - This is called alpha (α) level
  - α level usually selected are .01, or, 05, or .1

CRITICAL REGION



Finding the Critical Value for Specific α Values

- Normal distribution table to be used
- Step 1
  - Draw the figure and indicate the appropriate area



  - If the test is left-tailed, the critical region, with an area equal to α will be on the left side of the mean
    - For right sided test, it will be on right side
  - If the test is two-tailed, α must be divided by 2; one half of the area will be to the right of the mean, and one-half will be to the left of the mean

- **Contd. (finding critical …)**
- Step 2
  - For a left tailed test, use the z value that corresponds to the area equivalent to α in the Z-table
  - For a right- tailed test, use the z value that corresponds to the area equivalent to 1- α
  - For a two-tailed test, use the z value that corresponds to α/2 for the left value.
    - It will be negative
  - For the right value use the z value that corresponds to the area equivalent to 1- α/2
    - It will be positive

- ***Contd. (Errors …)***
- The probability of Type I error is denoted by the Greek alphabet α (*alpha*)
- The probability of Type II error is denoted by the Greek letter β *(beta)*
- The error probabilities of α and β are inversely related
- In the judicial system Type I error is more serious
- The probability is that many guilty people may escape conviction

- ***Contd. (Hypothesis …)***
- Step 3
- Testing hypothesis:
  - In statistics often hypothesis is tested about parameters

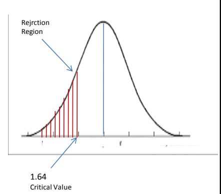
    A parameter is a numerical value that states something about the entire population being studied.

  - Hypotheses that are tested are generated by the questions that need to be answered
  - Example:
    - Average time required for admitting a patient is 5 hours after admission advice by the consultant
    - The Standard deviation is 1 hour
    - This is contested and claimed that it is much less
    - This is to be tested whether this claim is correct

- ***Contd. (Hypothesis …)***
- So, in this example:
  - H_0: 5 Hrs
  - H_1: <5 Hrs
- To test, a sample of 30 admission- time is recorded
  - To be tested at significance level (α) .05
  - This is a lower tail test (As in the H_1, the less than sign "< " points towards the left)
  - In the sample, the mean time for admission is 4.75
  - As the population SD is known and the sample size is 30, a Z score is calculated by the formula $Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$

The Formula is: $Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$

Where,
Z (zee) = Normal Distribution Statistic
X¯ (X bar) = Sample mean
μ (mu) = Population mean
n = Sample size
σ = (sigma) standard deviation



- The critical value at .05 level of significance found in z score table is 1.64
- Using the above formula the Z score of the problem is:

= -2.73861

As this score is more extreme than the critical value, the score is in the rejection region

Step 4

Decision making:

Hence, the null hypothesis is rejected

P-value Method for Hypothesis Testing

- There are several drawbacks to the rejection region method
  - Foremost among them is the type of information provided by the result of the test
  - Rejection region method produces a yes or no response to the question,
    - Is there sufficient statistical evidence to infer that the alternative hypothesis is true
- Hypotheses are usually tested at alpha levels of 0.05, 0.01 and sometimes at 0.10

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- ***Contd. (p-value …)***
- Choice of the level depends on how serious will be the type I error
- Many software packages are available for testing of hypothesis

> "The P-value (or probability value) is the probability of getting a sample statistic in the direction of the alternative hypothesis when the null hypothesis is true"

- P-value is the actual area under the standard normal distribution curve
  - Or, other curve, depending on type of statistical test

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- ***Contd. (P-value …)***
- Example:
  - Let Alternative hypothesis is H1: μ> 50, mean of the sample is X-bar = 52
  - Say, the computer output for p-value is 0.0356 for a statistical test,
    - then the probability of getting a sample mean of 52 or greater is 0.0356, if the true population mean is 50
      - (For the given sample size and SD)

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- ***Contd. (p-value …)***
  - The relationship between the p-value and the α can be explained as:
    - For P = 0.0356, the null hypothesis would be rejected at α = 0.05, but not at α = 0.01
    - *(The figure next slide illustrates the concept)*
- When the hypothesis test is two tailed, area in one tail must be multiplied by 2
- For a two tailed test , if α is 0.05, and the area in one tail is 0.0356, the p-value will be 2 x 0.0356 = 0.0712
  - That is, the null hypothesis should not be rejected at $\alpha = 0.05$ since 0.0712 is greater than 0.05

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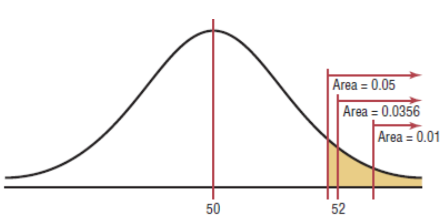
Relationship between α – value and *P*-value



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- ***Contd. (P-value …)***
- To sum up,
  - If the p-value is less than α , reject the null hypothesis
  - If the P-value is greater than α, do not reject the null hypothesis
- The P-value of Z-test can be found in Z table
  - First, find area under the standard normal distribution curve corresponding to the z test value
  - For a left-tailed test, use the area given in the table
  - For a right-tailed test, use 1.0000 minus the area given in the table.
  - For a two-tailed test, double the area found in the tail

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- ***Contd. (P-value …)***
- Decision Rule When Using a P-Value
  - If P-value ≤ α, reject the null hypothesis
  - If P-value > α do not reject the null hypothesis
- Describing the *P*-value
  - Following descriptive terms for p-values can be used:
  - If the p-value is:
    - less than .01, the test is highly significant
    - between ,01 and .05, it is significant
    - between .05 and .10 result is statistically not significant
    - Exceeding .10, there is little or no evidence to infer that the alternative hypothesis is true

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Student *t*-Test

- When population standard deviation is unknown, the z test is not normally used for hypothesis testing
- A different test, called *t-test* is used
  - The distribution of the variable should be approximately normal
- The *t-distribution* is similar to the standard normal distribution

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- ***Contd. (t distribution)***
- Similarity of *t* distribution with z distribution
  1. It is bell-shaped
  2. It is symmetric about the mean
  3. The mean, median and mode equal to 0 and are located at the centre of the distribution
  4. The curve never touches the x-axis
- Difference of t distribution from z distribution
  1. The variance is greater than 1
  2. The t distribution is a family of curves based on the degree of freedom
     - It is one less than the sample size (*df* = n-1)
  3. As the sample size increases, the t distribution approaches the normal distribution

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- ***Contd. (t test)***
- The formula of t test is similar to the formula for the z test
- As the population Standard Deviation is not known, sample SD is used
- The critical values of t test are found in t table
- For a one-tailed test find the alpha level by looking at the top row of the table and finding the appropriate column
- Find the degrees of freedom by looking at the left-hand column

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- For t-test same method as z test is used using the t table
- Steps:
  1. State the hypothesis and identify the claim
  2. Find the critical value(s) from t table
  3. Compute the test value
  4. Make the decision to reject or not reject the null hypothesis

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Where,
t = symbol of t test
X bar = sample mean
μ = population mean
n = sample size
S = sample standard deviation

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- ***Contd. (t test …)***
- Example:
  - The infection control team of a leading hospital reported the SSI rate as 16.3 per week
  - A random sample of 10 weeks had a mean of 17.7 SSIs. The sample SD is 1.8
  - Is there enough evidence to reject the investigator's claim at α = 0.05?
- Step 1 H_0:μ = 16.3 (claim) and H_1: ≠ 16.3
- Step 2 The critical values are + 2.262 and – 2.262
- Step 3 The test value t = 2.46
- Step 4 Reject the null hypothesis since 2.46>2.262
- Step 5 There is enough evidence to reject the claim that the average number of infections is 16.3

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Thank you